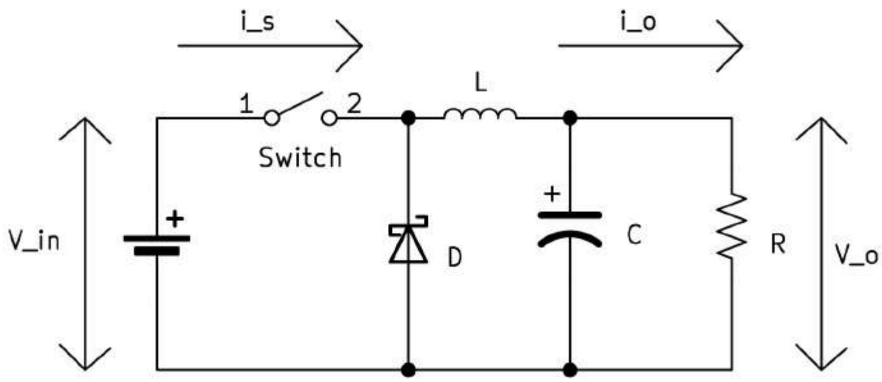
⚡

Buck Converter

🍀 ***INTRODUCTION:***



$Fig : Buck\ Converter$

Continuous Conduction Mode:

During ON time:

$$L\frac{di}{dt} = V_s - V_o$$

$$\Delta i = \frac{V_s - V_o}{L} T_{ON}$$

During OFF time:

$$L\frac{di}{dt} = -V_o$$

$$\Delta i = \frac{-V_o}{L}T_{OFF}$$

For the steady state operation Total current rise should be zero,

$$\Delta i_{ON} + \Delta i_{OFF} = 0$$

$$\frac{V_s - V_o}{L}T_{ON} + \frac{-V_o}{L}T_{OFF} = 0$$

$$\frac{V_s}{L}T_{ON} - \frac{V_o}{L}(T_{ON} + T_{OFF}) = 0$$

$$V_o = \frac{T_{ON}}{T_{ON} + T_{OFF}}V_s$$

$$V_o = D * V_s$$

$$where, D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$Also, D' = 1 - D$$

Now,

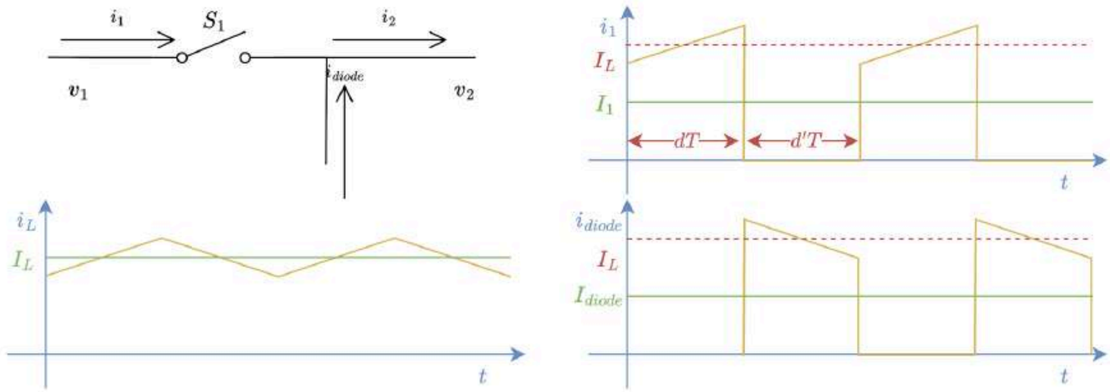
Considering Efficiency(η)= 100%

$$Power_{input} = Power_{output}$$

$$V_s * I_s = V_o * I_o$$

$$I_s = D * I_o$$

🍁 *Average modelling of switching devices*



$Fig: \ Average\ Modelling\ of\ switch\ with\ perturbation$

From above figure from averaging ignoring switching frequency ripple,

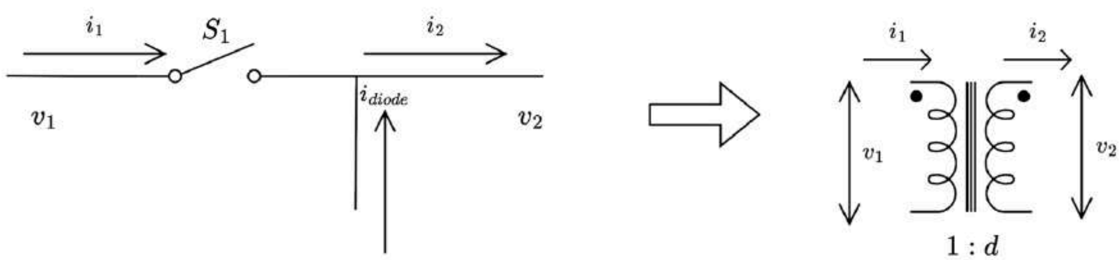
$$i_1 = d * i_2 \ can\ be\ written\ as,$$

$$i_1 = d * i_2 \ and\ I_1 = D * I_2 \ for\ DC\ (no\ AC\ perturbation)$$

Also, by equating Power,

$$v_2 = d * v_1 \ and\ V_2 = D * I_1 \ for\ DC\ (no\ AC\ perturbation)$$

Hence, the equivalent circuit is:



$Fig: Equivalent\ Circuit$

The transformer in figure is mathematical object which also works on DC. The transformer winding is which changes with time that makes this transformer non-linear. It is also large signal model

Small Signal Modelling of Switching Device

Lets decouple AC and DC part,

$i_1 = I_1 + \hat{i}_1$ $\quad and \quad$ $i_2 = I_2 + \hat{i}_2$

$$v_1 = V_1 + \hat{v}_1 \qquad and \qquad v_2 = V_2 + \hat{v}_2$$

$$d = D + \hat{d} \qquad and \qquad d' = 1 - d = 1 - (D + \hat{d}) = D' - \hat{d}$$

Now, From Average modelling we have,

$$i_1 = d * i_2$$

$$I_1 + \hat{i}_1 = (D + \hat{d}) * (I_2 + \hat{i}_2)$$

$$I_1 + \hat{i}_1 = DI_2 + \hat{d}I_2 + D\hat{i}_2 + \hat{d}\,\hat{i}_2$$

$$Since, I_1 = DI_2$$

$$\hat{i_1} = \hat{d}I_2 + D\hat{i_2} + \hat{d}\,\hat{i}_2$$

$$Here, \hat{i_1} << I_1,, \hat{i_2} << I_2 \; and \; \hat{d} << D \; so \; \hat{d}\hat{i_2} \rightarrow 0$$

$$\Rightarrow \; \hat{i_1} = \hat{d}I_2 + D\hat{i_2}$$

$$\Rightarrow \; \mathbf{\hat{i_1} = \hat{d}I_2 + D\hat{i_2}}$$

Similarly,

$$v_2 = dv_1$$

$$\Rightarrow (V_2 + \hat{v_2}) = (D + \hat{d})(V_1 + \hat{v_1})$$

$$\Rightarrow V_2 + \hat{v_2} = DV_1 + \hat{d}V_1 + \hat{d}V_1 + \hat{d}\hat{v_1}$$

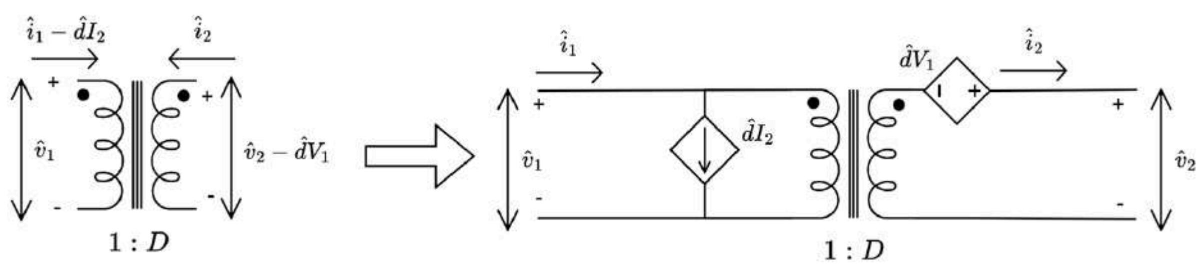
$$Since, V_2 = DV_1$$

$$\Rightarrow \hat{v_2} = \hat{d}V_1 + \hat{d}V_1 + \hat{d}\hat{v_1}$$

$$Here, \hat{v_1} << V_1, \hat{v_2} << V_2 \; and \; \hat{d} << D \; so \; \hat{d}\hat{v_1} \rightarrow 0$$

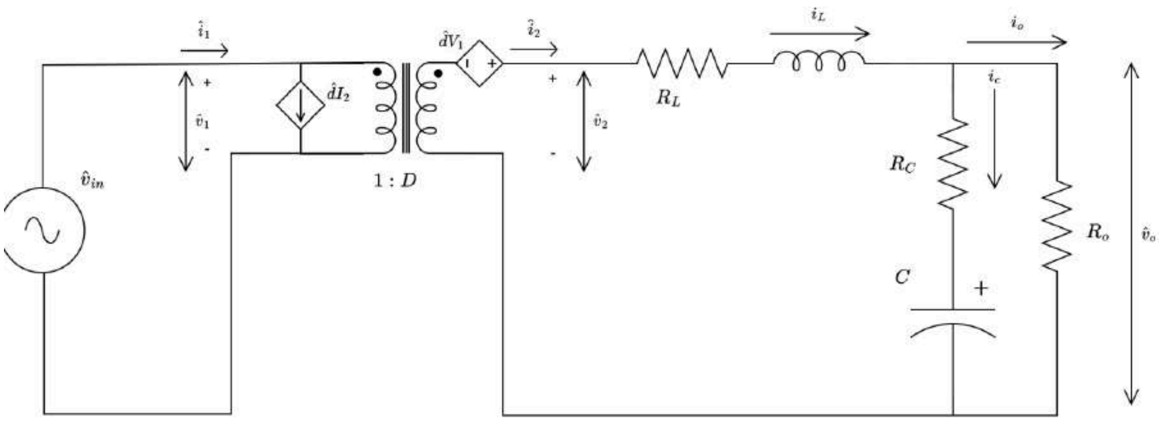
$$\Rightarrow \hat{v_2} = \hat{d}V_1 + D\hat{v_1}$$

$$\Rightarrow \mathbf{\hat{v_2} = \hat{d}V_1 + D\hat{v_1}}$$



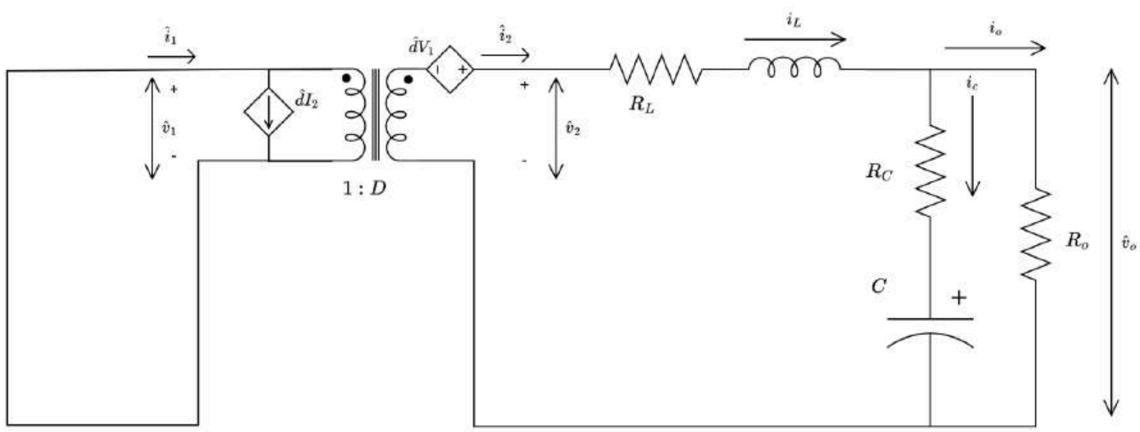
$$Fig : Equivalent \; Circuit \; for \; small \; signal \; analysis$$

🌱 Small Signal Complete Model



$$Fig : Small\ Signal\ Complete\ Model$$

📈 Gain Analysis:



$$Fig : Small\ signal\ model\ for\ gain\ analysis$$

From the figure in primary side,

$$\Rightarrow\ \hat{v}_1(s) = 0$$

From the figure in secondary side,

$$\Rightarrow\ \hat{v}_2(s) - (sL + R_L)\hat{i_L}(s) - \hat{v}_o(s) = 0$$

$$\Rightarrow\ \hat{v}_2(s) = (sL + R_L)\hat{i_L}(s) + \hat{v}_o(s)$$

Current through inductor,

$$\Rightarrow \hat{i_L}(s) = \hat{v_o}(s)(\frac{1}{R_o} + \frac{1}{R_C + \frac{1}{sC}})$$

$$\Rightarrow \hat{i_L}(s) = \hat{v_o}(s)(\frac{1}{R_o} + \frac{sC}{sR_CC+1})$$

$$\Rightarrow \hat{i_L}(s) = \hat{v_o}(s)\frac{1}{R_o}(1 + \frac{sR_oC}{sR_CC+1})$$

$$\Rightarrow \hat{i_L}(s) = \hat{v_o}(s)\frac{1}{R_o}\frac{s(R_o+R_C)C+1}{sR_CC+1}$$

From voltage relation,

$$\hat{v_2} - \hat{d}V_1 = D\hat{v_1}$$

$$\hat{v_2}(s) - \hat{d}(s)V_1 = D\hat{v_1}(s) \quad In\ Laplace\ domain$$

$$\Rightarrow (sL+R_L)\hat{i_L}(s) + \hat{v_o}(s) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow (sL+R_L)\hat{v_o}(s)\frac{1}{R_o}\frac{s(R_o+R_C)C+1}{sR_CC+1}) + \hat{v_o}(s) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)((sL+R_L)\frac{1}{R_o}\frac{s(R_o+R_C)C+1}{sR_CC+1}) + 1) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{1}{R_o}\frac{s^2(R_o+R_C)LC + sL + sR_L(R_o+R_C)C + R_L + sR_oR_CC + R_o}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{1}{R_o}\frac{s^2(R_o+R_C)LC + s(L + R_L(R_o+R_C)C + R_oR_CC) + R_L + R_o}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s(\frac{L}{R_L+R_o} + \frac{R_L(R_o+R_C)+R_oR_C}{R_L+R_o} + 1}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s(\frac{L}{R_L+R_o} + \frac{R_C(R_L+R_o)+R_LR_o}{R_L+R_o}C) + 1}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s(\frac{L}{R_L+R_o} + (R_C + \frac{R_LR_o}{R_L+R_o})C) + 1}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s\sqrt{\frac{R_0+R_C}{R_L+R_o}LC}(\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + (\frac{R_C\sqrt{R_L+R_o}}{\sqrt{R_o+R_C}} + \frac{R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}})\sqrt{\frac{C}{L}}) + 1}{sR_CC+1}) - \hat{d}(s)$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s\sqrt{\frac{R_0+R_C}{R_L+R_o}LC}(\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{R_CR_L+R_CR_o+R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}}\sqrt{\frac{C}{L}}) + 1}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \hat{v_o}(s)(\frac{R_L+R_o}{R_o}\frac{s^2\frac{R_o+R_C}{R_L+R_o}LC + s\sqrt{\frac{R_0+R_C}{R_L+R_o}LC}(\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{R_CR_L+R_CR_o+R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}\sqrt{\frac{L}{C}}}) + 1}{sR_CC+1}) - \hat{d}(s)V_1 = 0$$

$$\Rightarrow \frac{\hat{v_o}(s)}{\hat{d}(s)V_1} = \frac{R_o}{R_L+R_o}\frac{sR_CC+1}{s^2\frac{R_o+R_C}{R_L+R_o}LC + s\sqrt{\frac{R_0+R_C}{R_L+R_o}LC}(\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{R_CR_L+R_CR_o+R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}\sqrt{\frac{L}{C}}}) + 1}$$

$$DC\ quantities\ are:\ V_1 = V_s,\ V_2 = DV_s,\ I_L = I_2 = I_o = \frac{I_s}{D} = \frac{V_o}{R_o} = \frac{DV_s}{R_o}$$

$$\Rightarrow \frac{\hat{v}_o(s)}{\hat{d}(s)} = V_s \frac{R_o}{R_L + R_o} \frac{sR_C C + 1}{1 + +s\sqrt{\frac{R_0+R_C}{R_L+R_o}LC}\left(\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{R_C R_L + R_C R_o + R_L R_o}{\sqrt{(R_L+R_o)(R_o+R_C)}\sqrt{\frac{L}{C}}}\right) + s^2 \frac{R_o+R_C}{R_L+R_o}LC}$$

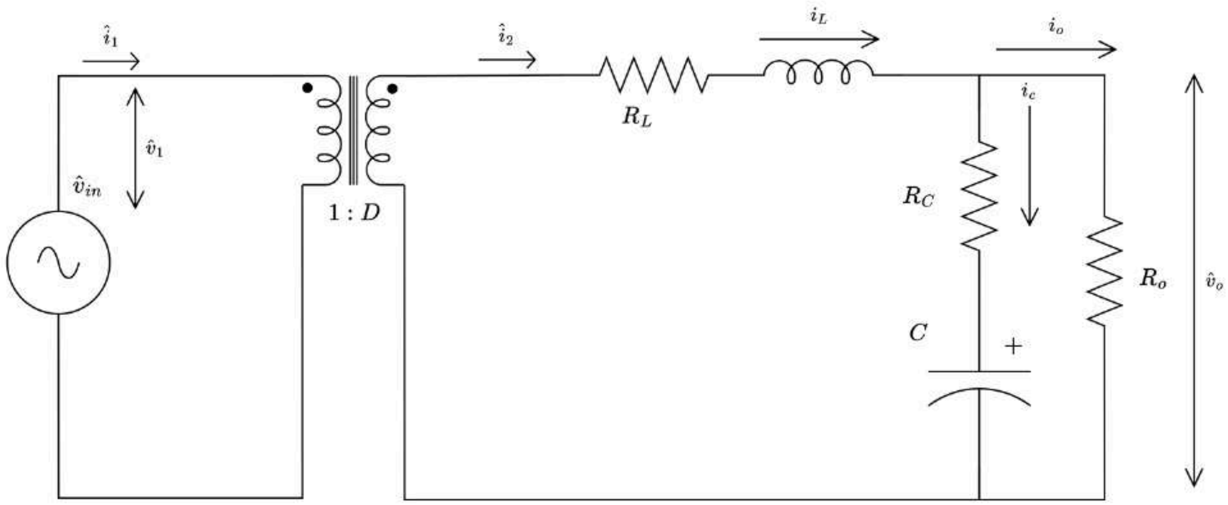
$$\Rightarrow G_{d2vo}(s) = G_{d2vo}(0) \frac{(a + \frac{s}{\omega_{esr}})}{1 + \frac{s}{\omega_o Q} + (\frac{s}{\omega_o})^2}$$

where,

$$G_{d2vo}(s) = V_s \frac{R_o}{R_L + R_o},\ \omega_{esr} = \frac{1}{R_C C},\ \omega_o = \frac{1}{\sqrt{\frac{R_o+R_C}{R_L+R_o}LC}}\ and$$

$$Q = \frac{1}{\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{\frac{R_C R_L + R_C + R_o + R_L R_o}{\sqrt{(R_L+R_o)(R_o+R_C)}}}{\sqrt{\frac{L}{C}}}}$$

📈 *Audio Susceptibility*



$$Fig:\ Small\ Signal\ Model\ for\ audio\ susceptibility$$

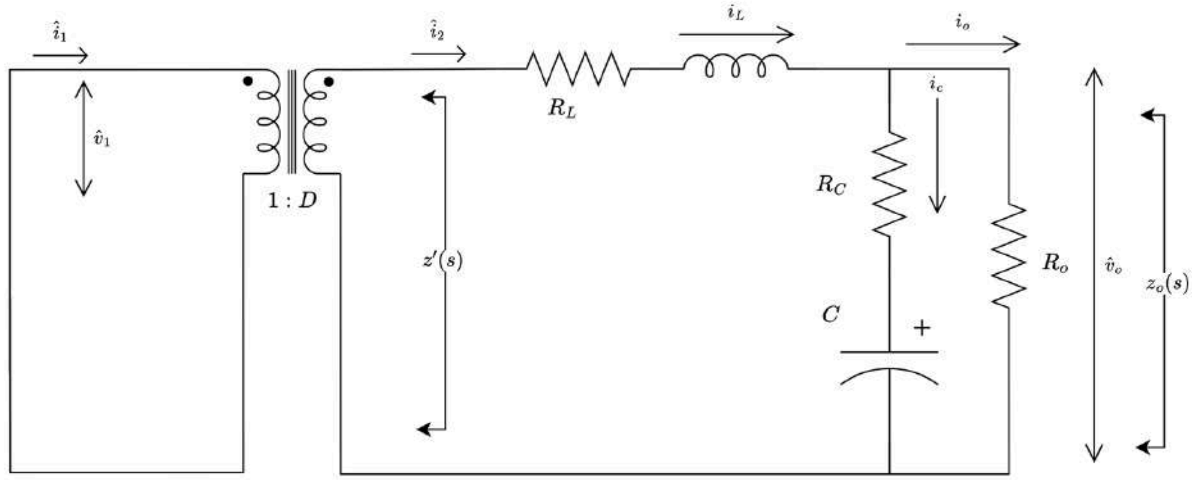
$$\Rightarrow G_{Vin2Vo}(s) = G_{Vin2Vo}(0) \frac{1 + \frac{s}{w_{esr}}}{1 + \frac{s}{\omega_o Q} + (\frac{s}{\omega_o})^2}$$

where,

$$G_{Vin2Vo}(0) = D \frac{R_o}{R_L + R_o},\ \omega_{esr} = \frac{1}{R_C C},\ \omega_o = \frac{1}{\sqrt{\frac{R_o+R_C}{R_L+R_o}LC}}\ and$$

$$Q = \frac{1}{\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{\frac{R_CR_L+R_C+R_o+R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}}}{\sqrt{\frac{L}{C}}}}$$

📈 *Output Impedance*



$Fig: \ Small \ Signal \ Model \ for \ Output \ Impedance$

$$z_o(s) = \frac{R_oR_L}{R_L + R_o}, \ w_{dcr} = \frac{R_L}{L}, \ w_{esr} = \frac{1}{R_CC}, \ w_o = \frac{1}{\sqrt{\frac{R_o+R_C}{R_L+R_o}LC}} \ and$$

$$Q = \frac{1}{\frac{\sqrt{\frac{L}{C}}}{\sqrt{(R_L+R_o)(R_o+R_C)}} + \frac{\frac{R_CR_L+R_C+R_o+R_LR_o}{\sqrt{(R_L+R_o)(R_o+R_C)}}}{\sqrt{\frac{L}{C}}}}$$

📈 *Analysis:*

ccm_buck_average_modelling_analysis.pdf