CCM Buck Converter Average Modelling Analysis

1. Bode Analysis with respect to duty cycle
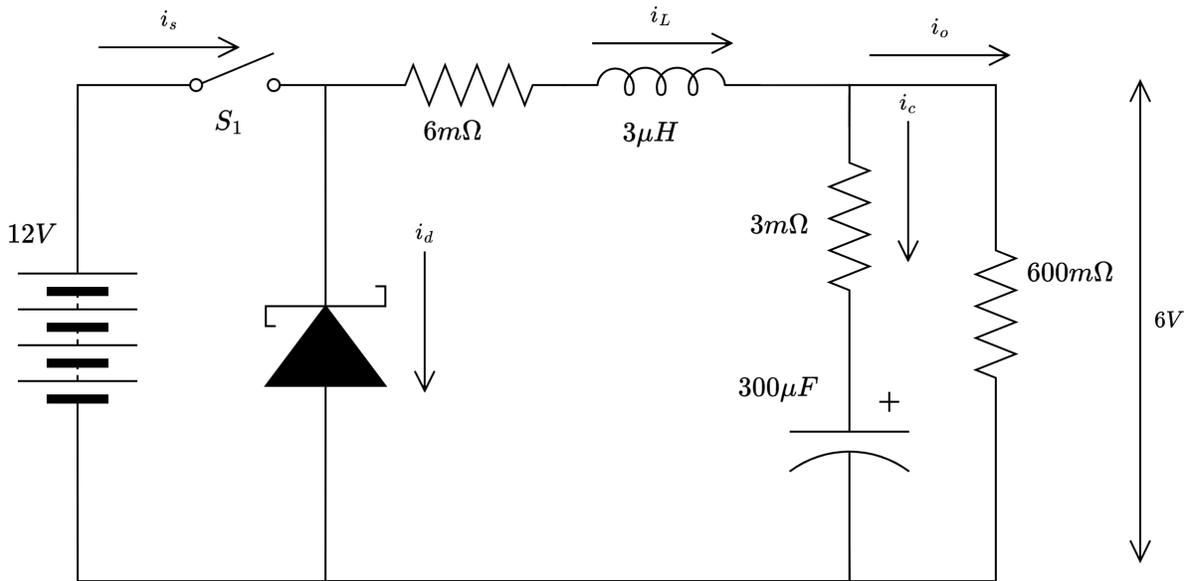

Fig. 8 Buck-Boost Converter

```matlab
clc;
clear;
close all;

% Drawing parameters
WIDTH = 900;
HEIGHT = 600;

syms s D
% Parameters
Vs = 12;
Vo = 6;
L = 3e-6;
Rl = 6e-3;
C = 300e-6;
Rc = 3e-3;
Fsw = 500e3;
Ro = 600e-3;
duty = Vo/Vs;% 0.5
```

$$\Rightarrow \mathbf{G_{d2vo}(s)} = \mathbf{G_{d2vo}(0)} \frac{\left(1 + \dfrac{s}{\omega_{esr}}\right)}{1 + \dfrac{s}{\omega_0 Q} + \left(\dfrac{s}{\omega_o}\right)^2}$$

$$\Rightarrow \mathbf{G_{vin2vo}(s) = G_{vin2vo}(0)} \frac{\left(1 + \dfrac{s}{\omega_{esr}}\right)}{1 + \dfrac{s}{\omega_0 \mathbf{Q}} + \left(\dfrac{s}{\omega_o}\right)^2}$$

$$\Rightarrow \mathbf{z_o(s) = z_o(0)} \frac{\left(1 + \dfrac{s}{\omega_{dcr}}\right)\left(1 + \dfrac{s}{\omega_{esr}}\right)}{1 + \dfrac{s}{\omega_0 \mathbf{Q}} + \left(\dfrac{s}{\omega_o}\right)^2}$$

where

$$\mathbf{G_{d2vo}(0) = V_s} \frac{\mathbf{R_o}}{\mathbf{R_o + R_L}}, \quad \mathbf{G_{vin2vo}(0) = D} \frac{\mathbf{R_o}}{\mathbf{R_o + R_L}}, \quad \mathbf{z_o(0) = } \frac{\mathbf{R_o R_L}}{\mathbf{R_L + R_o}},$$

$$\omega_{dcr} = \frac{\mathbf{R_L}}{\mathbf{L}}, \quad \omega_{esr} = \frac{1}{\mathbf{R_C C}}, \quad \omega_o = \frac{1}{\sqrt{\dfrac{\mathbf{R_o + R_C}}{\mathbf{R_L + R_o}} \mathbf{L\, C}}} \text{ and } \mathbf{Q} = \frac{1}{\dfrac{\sqrt{\dfrac{\mathbf{L}}{\mathbf{C}}}}{\sqrt{(\mathbf{R_L + R_o})(\mathbf{R_o + R_C})}} + \dfrac{\dfrac{\mathbf{R_C R_L + R_C R_o + R_L R_o}}{\sqrt{(\mathbf{R_L + R_o})(\mathbf{R_o + R_C})}}}{\sqrt{\dfrac{\mathbf{L}}{\mathbf{C}}}}}$$

```matlab
% Constants
Gd2vo_0 = Vs*Ro/(Ro+Rl);
Gvin2vo_0(D) = D*Ro/(Ro+Rl);
Zo_0 = Ro*Rl/(Ro+Rl);
w_dcr = Rl/L;
w_esr = 1/(Rc*C);
w_n = 1/sqrt(L*C);
w_o = w_n/sqrt((Ro+Rc)/(Ro+Rl));
Zc = sqrt(L/C);
k = 1/sqrt((Ro+Rc)*(Ro+Rl));
Q = 1/(Zc*k+(Rc*Rl+Rc*Ro+Rl*Ro)*k/Zc);

% Transfer functions
den(s,D) = 1+s/(w_o*Q)+(s/w_o)^2;
Gd2vo(s,D)=Gd2vo_0*(1+s/w_esr)/den(s,D);
Gvin2vo(s,D)=Gvin2vo_0(D)*(1+s/w_esr)/den(s,D);
Zo(s,D)=Zo_0*(1+s/w_dcr)*(1+s/w_esr)/den(s,D);

Ds = linspace(0.01,0.99,5);
omegaRange = {1e2,1e6};

figure('Position',[0 0 WIDTH HEIGHT]);
hold on
legendString = "";
for v = Ds
    f(s) = vpa(simplify(Gd2vo(s,v)));
    legendString = legendString + "D = "+v+",";
    plotBode(f(s),omegaRange,{'Gain (Gd2vo)'});
```
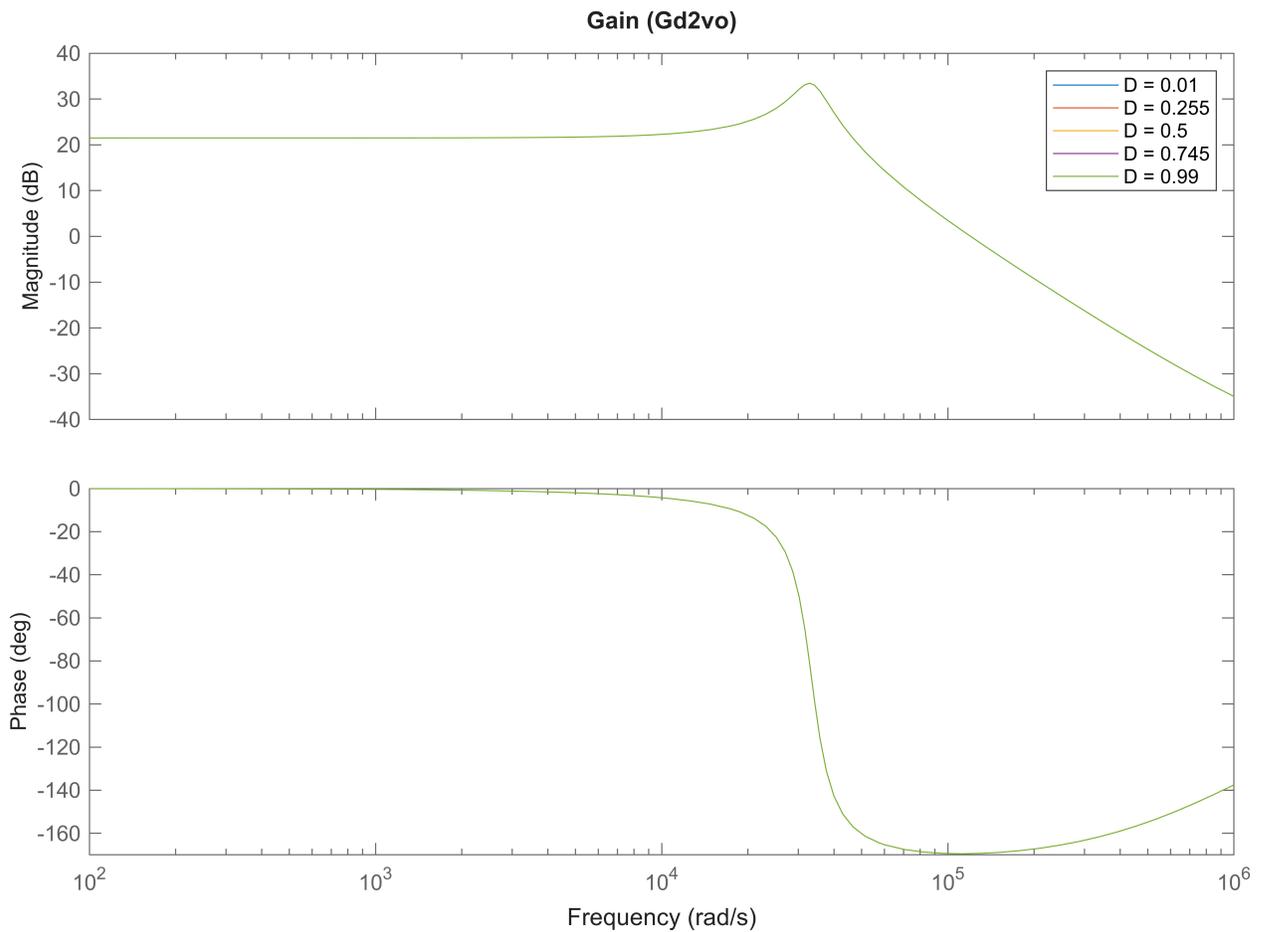
```
end
legendArr = split(legendString,",");
legend(legendArr(1:end-1));
hold off
```
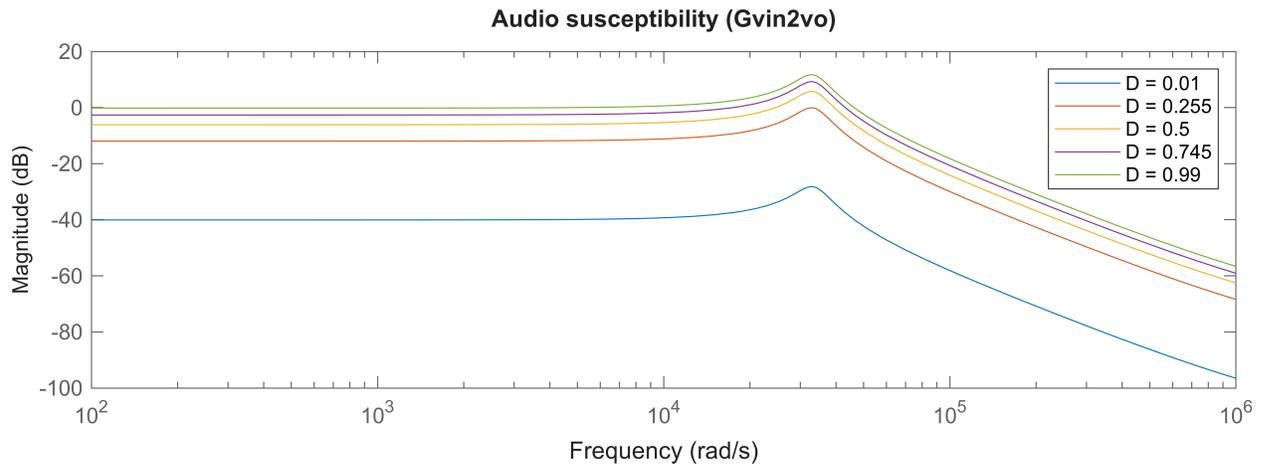
Gain (Gd2vo)


```
figure('Position',[0 0 WIDTH HEIGHT/2]);
hold on
legendString = "";
for v = Ds
    f(s) = vpa(simplify(Gvin2vo(s,v)));;
    legendString = legendString + "D = "+v+",";
    plotBode(f(s),omegaRange,{'Audio susceptibility (Gvin2vo)'},"off");
end
legendArr = split(legendString,",");
legend(legendArr(1:end-1))
hold off
```

Audio susceptibility (Gvin2vo)
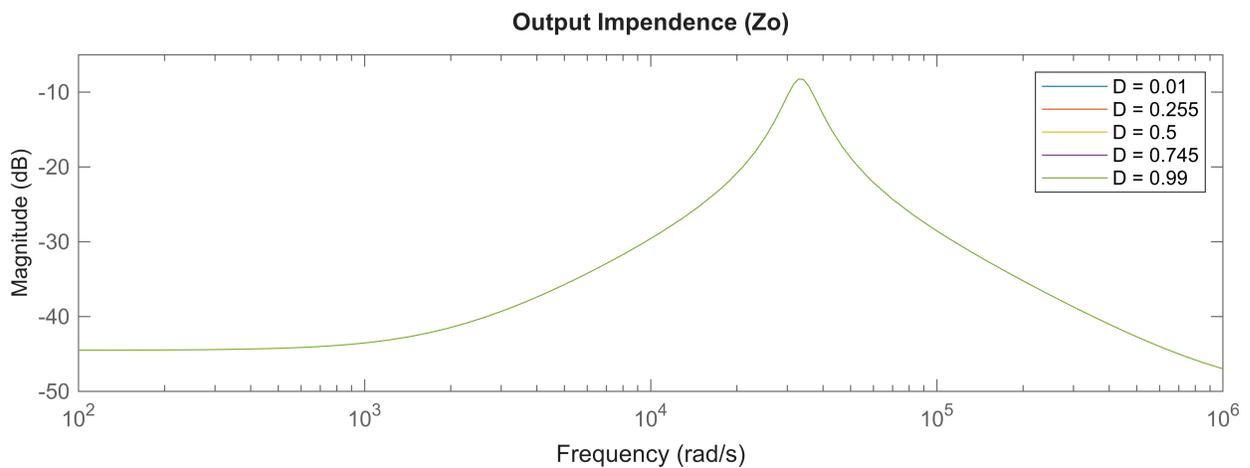


```
figure('Position',[0 0 WIDTH HEIGHT/2]);
hold on
legendString = "";
for v = Ds
    f(s) = vpa(simplify(Zo(s,v)));
    legendString = legendString + "D = "+v+",";
    plotBode(f(s),omegaRange,{'Output Impendence (Zo)'},"off");
end
legendArr = split(legendString,",");
legend(legendArr(1:end-1))
hold off
```

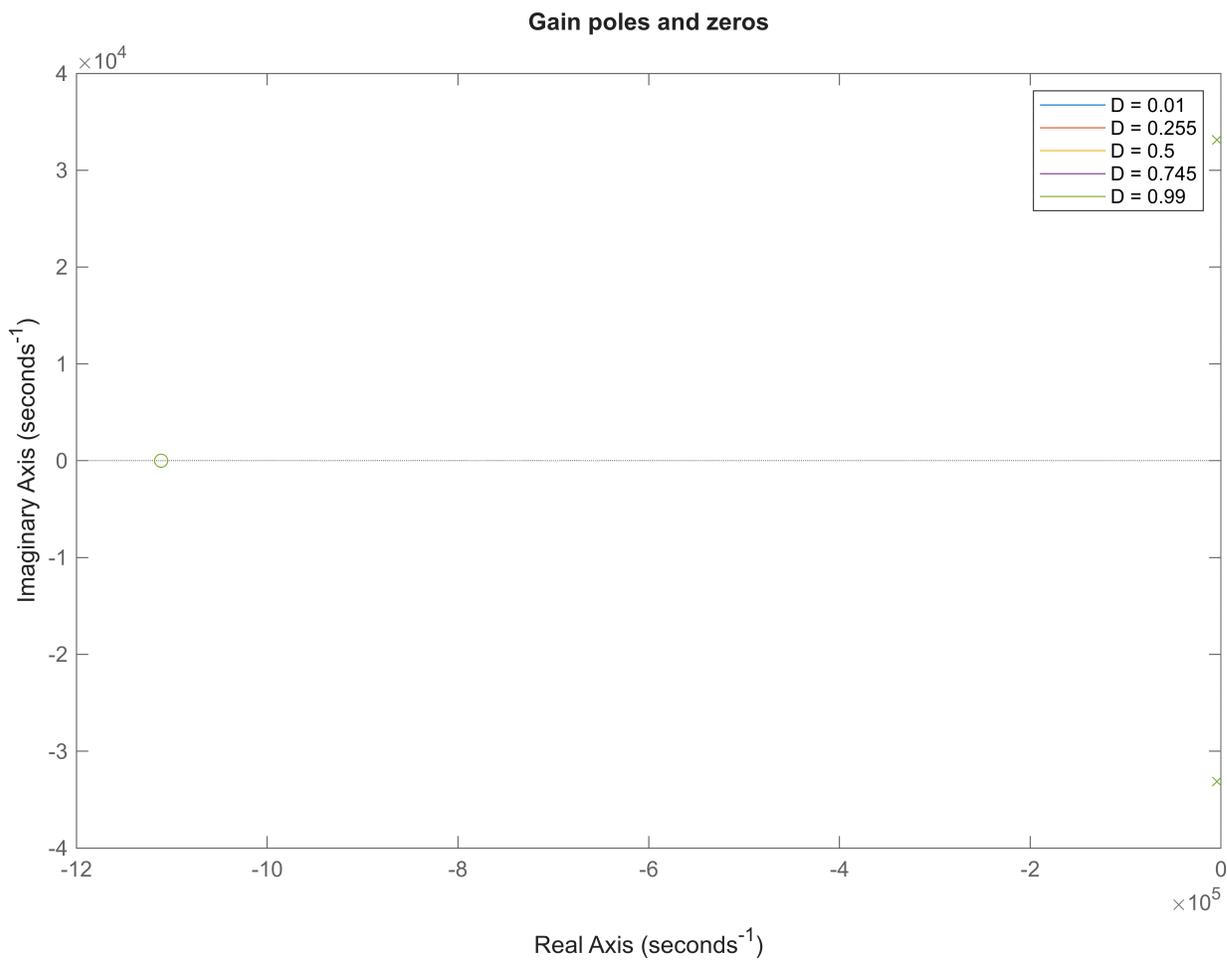## Output Impendence (Zo)


```
figure('Position',[0 0 WIDTH HEIGHT]);
hold on
legendString = "";
for v = Ds
    f(s) = vpa(simplify(Gd2vo(s,v)));
    legendString = legendString + "D = "+v+",";
    plotZP(f(s),{'Gain poles and zeros'});
```

```
end
legendArr = split(legendString,",");
legend(legendArr(1:end-1))
hold off
```

Gain poles and zeros


```
function plotBode(G_s,wRange,title,phaseVisible)
    arguments
        G_s
        wRange = {1,1e5}
        title = {}
        phaseVisible = "on"
    end

    [n,d] = numden(G_s);
    nArr = sym2poly(n);
    dArr = sym2poly(d);
    model = tf(nArr,dArr);
    bp = bodeplot(model,wRange);
    if ~isempty(title)
```

```matlab
        bp.Title.String = title;
    end
    bp.PhaseVisible = phaseVisible;
end

function plotZP(G_s,title)
    arguments
        G_s
        title = {}
    end

    [n,d] = numden(G_s);
    nArr = sym2poly(n);
    dArr = sym2poly(d);
    model = tf(nArr,dArr);
    pp = pzplot(model);
    if ~isempty(title)
        pp.Title.String = title;
    end
end
```